Paired t-test

- **The paired sample *t*-test has two competing hypotheses, the null hypothesis and the alternative hypothesis.**

- **The null hypothesis assumes that the true mean difference between the paired samples is zero.**

- **The alternative hypothesis can take one of several forms depending on the expected outcome. If the direction of the difference does not matter, a two-tailed hypothesis is used. Otherwise, an upper-tailed or lower-tailed hypothesis can be used to increase the power of the test.**

Summary for Paired t-test

The paired sample *t*-test hypotheses are formally defined below:

- The null hypothesis (H0) assumes that the true mean difference (μd) is equal to zero.
- The two-tailed alternative hypothesis (H1) assumes that μd is **not equal** to zero.
- The upper-tailed alternative hypothesis (H1) assumes that μd is **greater** than zero.
- The lower-tailed alternative hypothesis (H1) assumes that μd is **less** than zero.

The mathematical representations of the null and alternative hypotheses are defined below:

$H_0: \mu d = 0$

$H_1: \mu d \neq 0$ (two-tailed)

$H_1: \mu d > 0$ (upper-tailed)

$H_1: \mu d < 0$ (lower-tailed)

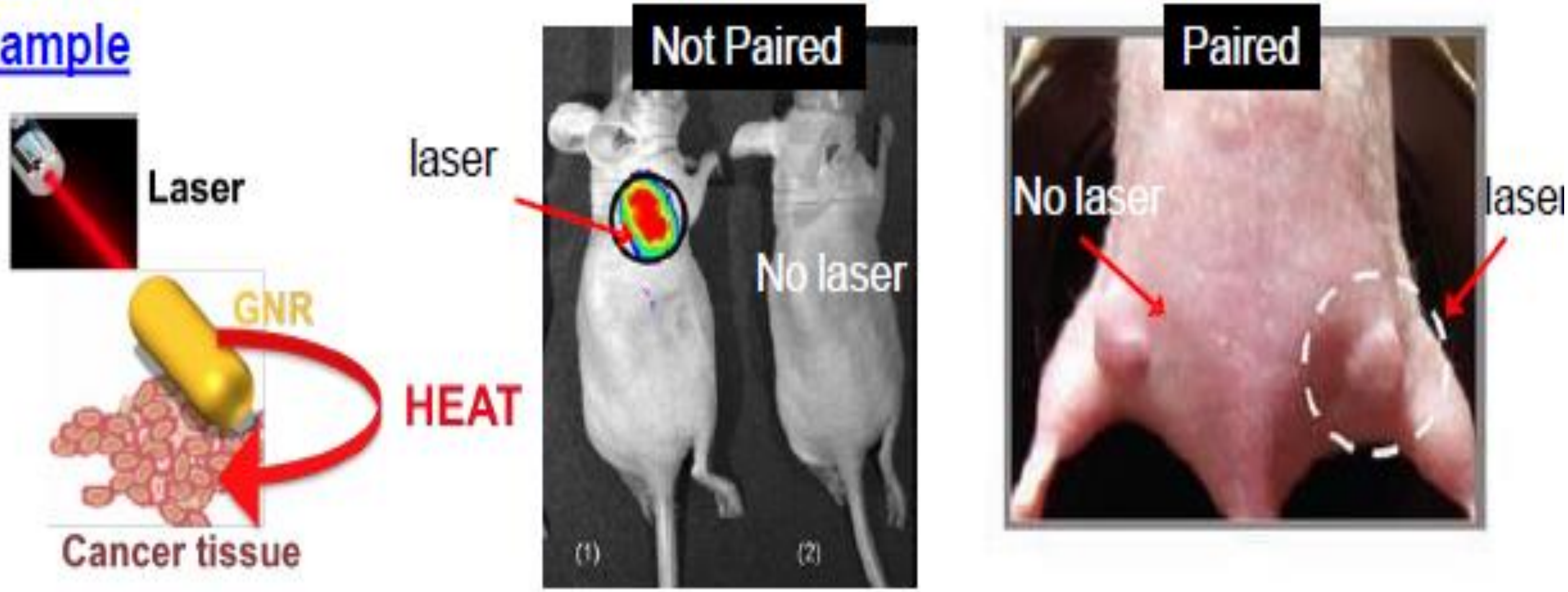
Note. It is important to remember that hypotheses are never about data, they are about the processes which produce the data. In the formulas above, the value of μ_d is unknown. The goal of hypothesis testing is to determine the hypothesis (null or alternative) with which the data are more consistent.

(Paired Comparisons: Paired t-test)

- The objective in paired comparison tests is to eliminate a maximum number of sources of extraneous variations by making the pairs similar with respect to as many variables as possible.

- Related or paired observations may be obtained in a number of ways:
  - The same subjects may be measured before and after receiving some treatment.
  - In comparing two methods of analysis, the material to be analyzed may be divided equally so that one half is analyzed by one method and one half is analyzed by another.

Example



To study the effect of gold nanoparticles in treating tumors, we induce tumor and then target it with gold nanoparticles which serve as "nanoheaters" upon radiation with laser. In "Not Paired" case, we use two groups that one receives gold nanoparticles and the other does not (control). The difference between treated and control may be due simply a difference in external characteristics between the mice in both groups. To eliminate this difference, we can use one group of mice and induce two identical tumors in the same mouse as you see in the picture to the right. Or we can use the same mice with before/after strategy.

Paired Comparisons

- Instead of performing the analysis with individual observations, we use d_i, the difference between pairs of observations as the variable of interest.

Paired Comparisons

- When the n sample differences computed from the n pairs of measurements constitute a simple random sample from a normally distributed population of differences, the test statistic for testing hypothesis about the population mean difference μ_d is:

$$t = \frac{\bar{d} - \mu_{d_0}}{SE}$$

where :

$\bar{d}$ is the sample mean difference

μ_{d_0} is the hypothesized population mean difference

$$SE = \frac{s_d}{\sqrt{n}}$$

n is the number of sample differences

s_d is the standard deviation of the sample differences

Paired Comparisons

- The t statistic is distributed as Student's t with n-1 degrees of freedom.

- We do not have to worry about the equality of variances in paired comparisons, since our variable is the difference in the reading of the same subject or object.

Steps for Calculating Paired Sample *t* Tests

- Step 1: Identify the populations, distribution, and assumptions.
- Step 2: State the null and research hypotheses.
- Step 3: Determine the characteristics of the comparison distribution.
- Step 4: Determine critical values, or cutoffs.
- Step 5: Calculate the test statistic.
- Step 6: Make a decision.

Paired Comparisons

- In a study to evaluate the effect of very low calorie diet (VLCD) on the weight of 9 subjects, the following data was collected:

B (before)	117.3	111.4	98.6	104.3	105.4	100.4	81.7	89.5	78.2
A (After)	83.3	85.9	75.8	82.9	82.3	77.7	62.7	69	63.9

- The researchers wish to know if these data provide sufficient evidence to allow them to conclude that the treatment is effective in causing weight reduction in those individuals.
- If we choose (d_i=A – B), the differences are:-34, -25.5, -22.8, -21.4, -23.1, -22.7, -19, -20.5, -14.3.

- Assumptions: the observed differences constitute a simple random sample from a normally distributed population of differences that could be generated.

Paired Comparisons

- We may obtain the differences in one of two ways: by subtracting the before weights from the after weights (A – B) or by subtracting the after weights from the before weights (B – A).
- If the test is two sided and the question of interest is there a difference in mean body weight: A-B or B-A can be used H_0 and H_a are the same for either:

$$\text{T-criticals} = \pm t(1-\alpha/2, df=n-1)$$

$$H_0: \mu_d = 0$$

$$H_A: \mu_d \neq 0$$

- If the question of interest does the VLCD result is significant weight reduction, H_0 and Ha change on whether A-B or B-A is used as follows

A-B	B-A
H_0 $\mu_d \geq 0$, H_a $\mu_d < 0$	$H_0 \leq 0$, $H_a > 0$
t-critical = $-t(\alpha, df=n-1)$ or t-critical = $-t(1-\alpha, df= n-1)$	t-critical = $t(1-\alpha, n-1)$

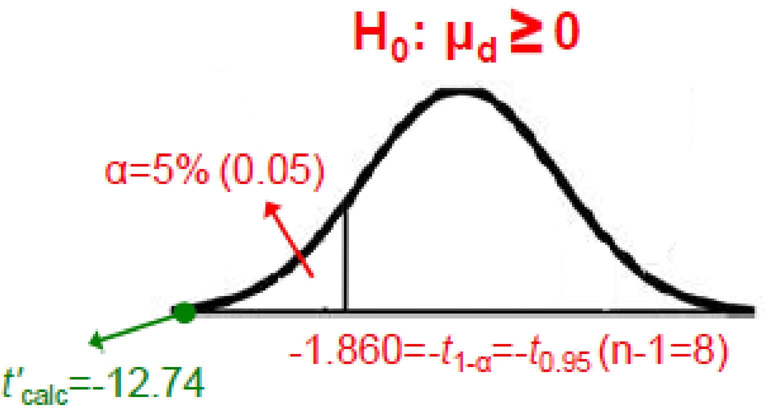
Paired Comparisons

- The test statistic: $$t = \frac{\bar{d} - \mu_{d_0}}{s_{\bar{d}}}$$
- Decision rule: Let α=0.05, and the question of interest was is their significant weight reduction after VLCD (Based on A-B H_0: $\mu_d > 0$; H_a: $\mu_d \leq 0$, left sided) the critical value of $t_{\alpha,\ df=8}$ or $-t_{1-\alpha/2,\ df=8}$ is -1.86, reject H_0 if the computed t is less than or equal to the critical value.

$$\bar{d} = \frac{\Sigma d_i}{n} = \frac{-203.3}{9} = -22.5889$$

$$s_d^2 = \frac{\Sigma(d_i - d)^2}{n-1} = 28.2961$$

$$t = \frac{-22.5889 - 0}{\sqrt{\frac{28.2961}{9}}} = -12.7395$$



Reject H_0, since -12.7395 is in the rejection region.

We may conclude that the diet program is effective

Paired Comparisons

- A 95% confidence interval for μ_d may be obtained as follows:

$$\bar{d} \pm t_{(1-\alpha),df=8} * SE$$

$$-22.5889 \pm 1.86\sqrt{28.2961/9}$$

$$-22.5889 \pm 4.0888$$

$$-26.68, -18.50$$