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لجان الدفعات



تفريغ إحصاء صيدلاني

hypothesis test

موضوع المحاضرة:

16

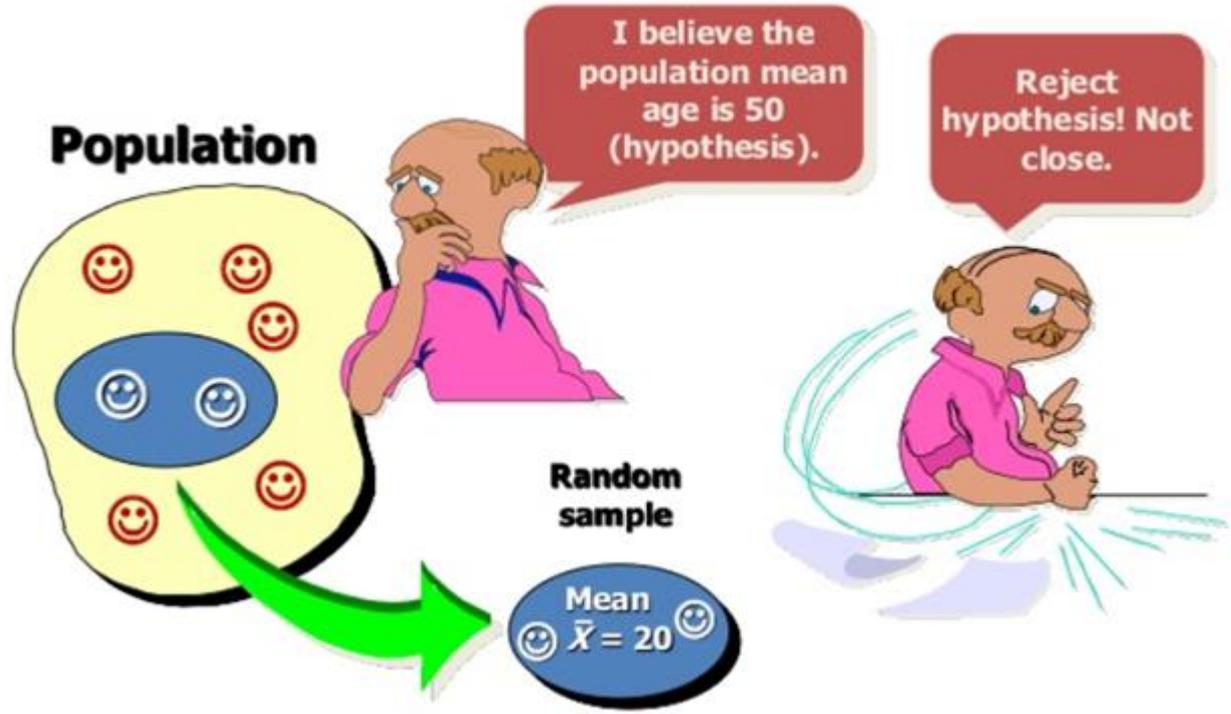
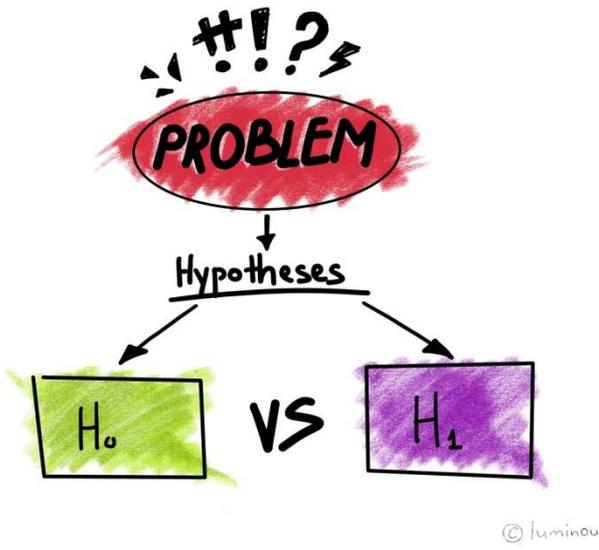
رقم المحاضرة:

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إعداد الصيدلانية:



Hypothesis Testing



- Hypothesis testing يكون في باحث عنده نظرية معينة ويجاوب يثبتها عن طريق ال statistics ، مثلاً يدك تبحث بدوا هذا كباحد في الشركة الصيدلانية ومطلوب منك مع مجموعة باحثين انك تثبت انه الدواء الجديد الي مهنعيه هل هو اكثر فاعلية من الادوية الموجودة في السوق
- النظرية الي دائماً موجودة وما فيها فرق او علاقة اسمها **null hypothesis**
- النظرية الي انت كباحد قدرت تثبتها اسمها **research or alternative hypothesis** ورفضها بالرمز h_1 او h_a
- يعني الباحث اما يقدر يثبت نظريته او ما يقدر يثبتها
- h_0 الي تبكي انه النظرية ما فيها فرق وما فيها علاقة اما منحي
- we can't reject h_0 يعني h_0 we accept يعني انت ما قدرت تثبت نظريتك او منحي
- we reject h_0 يعني h_1 we accept يعني قدرت تثبت نظريتك
- هاي النتائج ما منوطها الا عن طريق استخدام equation بالاعتماد على ال Z و T-table وعن خلالها فنقرر اذا بيم رفض او عدم رفض النظرية

Hypothesis Testing

- The general goal of a hypothesis test is to rule out chance (sampling error) as a plausible explanation for the results from a research study.
- Hypothesis testing is a technique to help determining whether a hypothesis is true (e.g. treatment, procedure has an effect in a population), or simply if a relationship exists between two or more variables.

• hypothesis testing بدنا نثبت انه في فرق بالبحق عن ال population و يكون not by chance يعني الفرق هاد يكون عن طريق دراسة علمية

• null hypothesis نتكفي انه حتى لو في فرق يكون by chance وانه ما في علاقة و ما في فرق

Research Hypothesis vs Null Hypothesis

Research hypothesis (H_1) is what the research believes to be a true reflection on the general population. In another word, a true explanation for a phenomena in the population. The researcher wants to prove that his sample statistics is different than the population parameters. Research hypothesis is also called alternative hypothesis.

Null hypothesis (H_0) is the opposite of H_1 . The H_0 assumes no difference of test statistics and the population parameter. This means that the researcher hypothesis about a certain phenomena is not correct, and that there is no real difference between sample and population for a certain feature or difference is due another reason (that is not tested).

Null & Alternative Hypothesis

Null Hypothesis or the Hypothesis to be Tested (H_0)

- A claim that there is NO difference between the population parameter like mean (μ) and the hypothesized value. In the testing process the null hypothesis either is rejected or not rejected (we do not say accepted).

Alternative Hypothesis or the Research Hypothesis (H_A or H_1)

- A statement of what we will believe is true if our sample data cause us to reject the null hypothesis. Usually the alternative hypothesis is the research hypothesis (the conclusion that the researcher is seeking to reach).

Null Hypothesis as an Assumption to be Challenged

- We might begin with a belief or assumption that a statement about the value of a population parameter is true.
- We then using a hypothesis test to challenge the assumption and determine if there is statistical evidence to conclude that the assumption is incorrect.
- In these situations, it is helpful to develop the null hypothesis first.

• اذا بدى اثبت نظرية لازم تكون clear يعنى انه انا ما بحط question و بحط عبارة واضحة clear statement

Example

- A new drug is developed with the goal of lowering blood pressure more than the existing drug.

- **Null hypothesis**

• null hypothesis يتحكي انه ما في فرق و ما في علاقة

- The new drug does not lower BP more than the existing drug.

- **Alternative hypothesis**

- The new drug lowers blood pressure more than the existing drug.

• more than يعني الـ μ تاعها اكبر من الـ μ تاع الـ population و انا انا انا اكثر للعين

• less than يعني الـ μ اقل من الـ μ تاع الـ population و انا انا انا اكثر للبيان

• لو انا عدي new drug و الهدف منه انه يقل lowering of blood pressure اكثر فعالية من الادوية الموجودة في السوق

Null & Alternative Hypothesis

Gabapentin has **no pharmacological effect**

- The mean for population A is 20 ($H_0: \mu = 20$)
- The mean for population A is less than 20 ($H_0: \mu \leq 20$)
- The mean for population A is larger than 20 ($H_0: \mu \geq 20$)

Gabapentin has **a pharmacological effect**

- The mean for population A is not 20 ($H_1: \mu \neq 20$)
- The mean for population A is larger than 20 ($H_1: \mu > 20$)
- The mean for population A is less than 20 ($H_1: \mu < 20$)

**Accepting or rejecting a hypothesis is not a proof of the hypothesis!
Null hypothesis can be true or false, we only can reject it or not to reject it**

research hypothesis اور null hypothesis کے درمیان

Null Hypothesis **vs** Alternative Hypothesis



Null hypothesis H_0	Alternative Hypothesis H_1
There is no relationship or difference	There is a relationship or difference
Refers to the population	Refers to the examined sample
Research aims to reject the null	Research aims to accept the alternative
Represent an original assumption	Prove statistically a systemic difference or relationship
Assumes a difference is due to chance	Assumes that difference is less likely to be due to chance.

Types of Statistical Errors in Hypotheses Testing

Because hypothesis tests are based on sample data, we must allow for the possibility errors.

Type I error

- A type I error is rejecting H_0 when it is true.
- The probability of making a Type I error when the null hypothesis is true as an equality is called the level of significance (α).

• پس اعرف انه type I هي α

Type II error

type II هي β

- A type II error is accepting H_0 when it is false.
- It is difficult to control for the probability of making a type II error (β).
- Statisticians avoid the risk of making a type II error by using “do not reject H_0 ” and not “accept H_0 ”

Errors in hypothesis testing

قرنی مفلوب

TRUTH

DECISION

	Ho is true ($A=B$)	Ho is false ($A \neq B$)
Reject Ho ($A \neq B$)	Type I error “giving a treatment that does not work”	Correct
Do not reject Ho ($A=B$)	Correct	Type II error “not giving a treatment that works”

How to establish a good hypothesis?

- Clear and declarative statement. Not a question.
- Show a relationship between variables
- Reflect a body of literature or a theory
- Be direct, explicit, and to the point.
- Be testable and measurable.

Critical Value Approach to Hypotheses Testing

مثلا $H_0 = 30$ و النظرية انه $H_0 \neq 30$ بين انا اكبر او اصغر

- **Step 1:** develop the null and alternative hypotheses.
- **Step 2:** specify the level of significance α . Common value of α are: $\alpha = 1\%$ or 0.01 , 5% or 0.05 , 10% or 0.1 اذا ما عدد بالوأل يستخدم 0.025
- **Step 3:** collect the sample data and compute the value of the test statistics Z or t.
- **Step 4:** use the level of significance (α) to determine the critical value (tabulated value). بعدت حسب ال calculated عن طريق لقانون
- **Step 5:** use the suitable rejection rule.
- **Step 6:** state the appropriate conclusions.

• في قيمتين ل Z
tabulated value (1) بطلها من الجدول
calculated value (2) بحسبها عن طريق
$$\frac{\bar{X} - \mu}{\sigma}$$

بعدت بقارنهم مع بعض

Step 1

- Set your hypothesis

Set both H_0 & H_1 .

Example

no longer (\neq)

It is believed that a candy machine makes chocolate bars that are on average 5g. A worker claims that the machine after maintenance no longer makes 5g bars. Write H_0 and H_a .

Answer:

$$H_0: \mu = 5$$

$$H_1: \mu \neq 5$$

A company has stated that their straw machine makes straws that are 4 mm diameter. A worker believes the machine no longer makes straws of this size and samples 100 straws to perform a hypothesis test with 99% confidence.

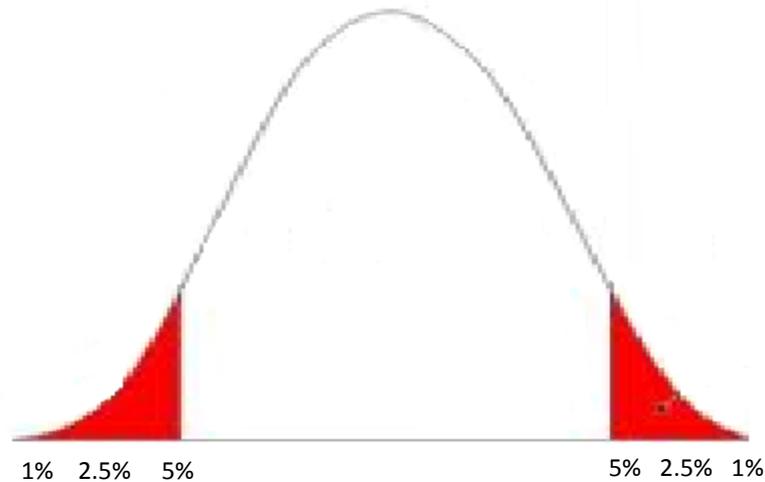
Answer:

$$H_0: \mu = 4$$

$$H_1: \mu \neq 4$$

Step 2

- Set level of significance associated with the hypothesis.



Step 3

(tabulated value)

- Set the critical value needed to reject the null hypothesis (from Tables).
- You might need the table of Z-test, t-test, F-test. *F test من جدول*
- Based on the chosen table, look for the cut off value based on the level of significance you determined in step 2.

known σ و normal distributed σ ليا Z -table

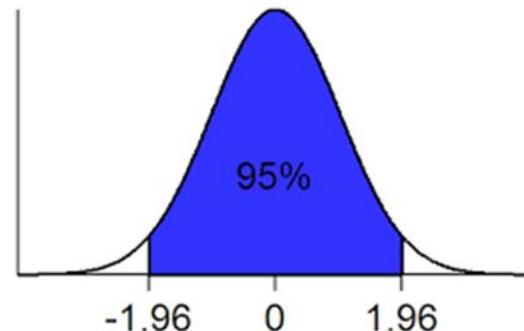
Z -table

unknown σ و normal distributed σ ليا T -table

T -table

unknown σ و not normal distributed σ ليا Z -table

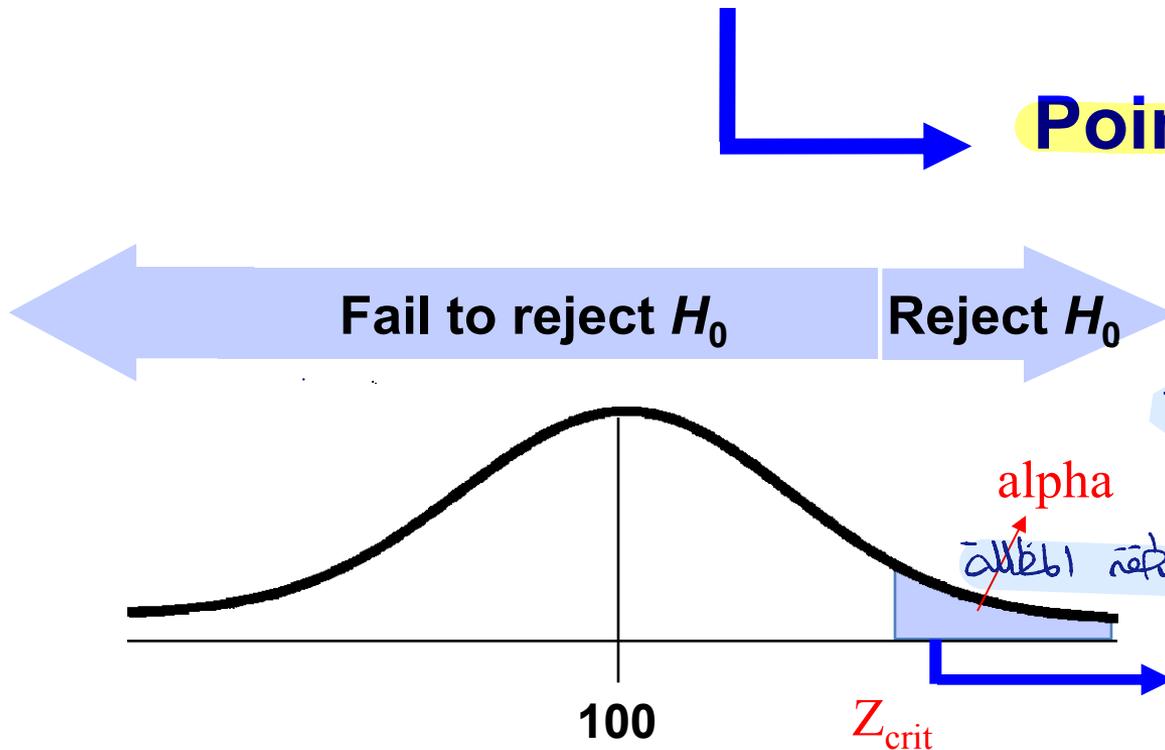
Z -table



Right-tailed tests

$$H_0: \mu = 100$$

$$H_1: \mu > 100$$



Points Right

إذا كانت النظرية $\mu = 100$ وبدي
أثبت أنه $\mu > 100$ يعني راج تكون
جهة اليمين فال α لل Z_{crit}
راج يكون موقعه على جهة اليمين
وليعني تحول $\bar{X} \leftarrow Z$. إذا ال
التي حسبها يعرف من ال Z_{crit}
معان راج تكون بالمنطقة الغير
مظللة وإذا كانت أكبر راج تكون بالمنطقة المظللة

Values that
differ "significantly"
from 100

Left-tailed tests

$$H_0: \mu = 100$$

$$H_1: \mu < 100$$

Points Left

$$z_{calc} < z_{crit}$$

صحة توقع في النتيجة المطلوبة

Reject H_0

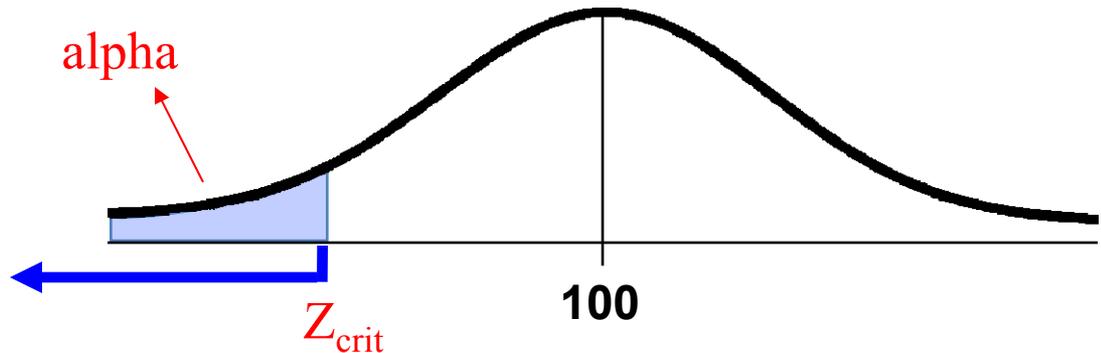
Fail to reject H_0

alpha

Values that differ "significantly" from 100

Z_{crit}

100



Two-tailed hypothesis testing

- H_A is that μ is *either* greater or less than μ_{H0}

$$H_A: \mu \neq \mu_{H0}$$

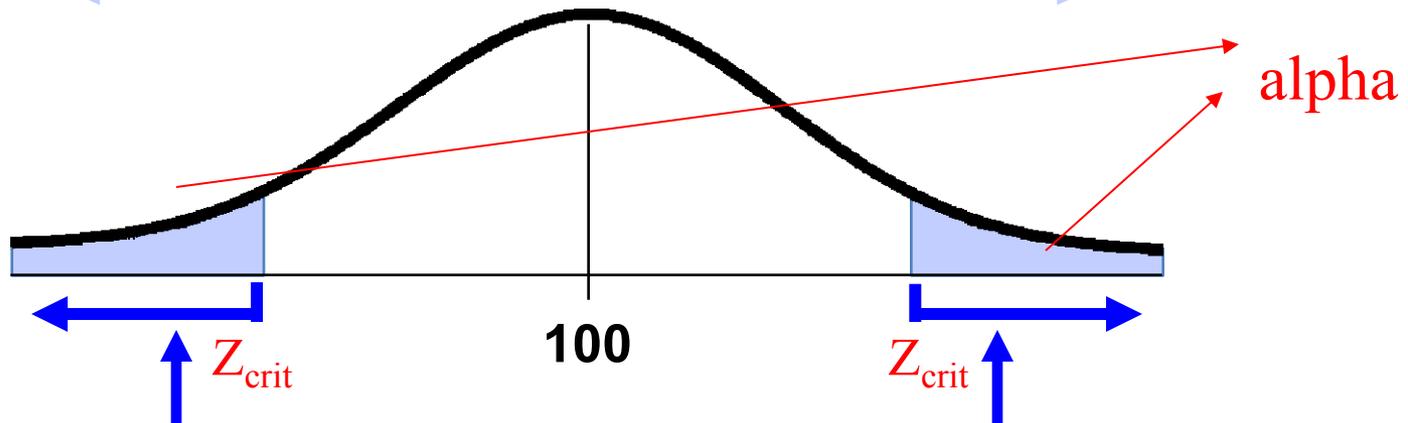
- α is divided equally between the two tails of the critical region

Two-tailed hypothesis testing

$$H_0: \mu = 100$$

$$H_1: \mu \neq 100$$

Means less than or greater than



Values that differ significantly from 100

One tale critical values

alpha .05, $Z_{\text{crit}}=1.64$;

alpha .01, $Z_{\text{crit}}=2.33$

يطلعهم من الحدود

Two tale critical values

alpha .05, $Z_{\text{crit}}=1.96$;

alpha .01, $Z_{\text{crit}}=2.58$

Normal Distribution

(Population Standard Deviation σ is known
or Standard Deviation σ is known but n is Large)

Normal Distribution

σ is known

[$n < 30$ (small) or $n \geq 30$ (large)]

$$z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Normal (σ is unknown) or unknown or non-normal distribution
but $n \geq 30$ (large)

$$z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

Normal Distribution

(Population Standard Deviation σ is Unknown and n is small)

Normal Distribution

σ is unknown

$N < 30$ (small)

$$t = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

Summary of Forms for Null and Alternative Hypotheses about a Population Mean

The equality part of the hypotheses always appears in the null hypothesis H_0 . In general, a hypothesis test about the value of a population mean μ must take one of the following three forms (where μ_0 is the hypothesized value of the population mean).

a) One – tailed test

i. $H_0: \mu = \mu_0$ vs $H_1: \mu < \mu_0$ (less than or smaller than)

ii. Upper (right)-tailed test

$H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$ (more than or greater than)

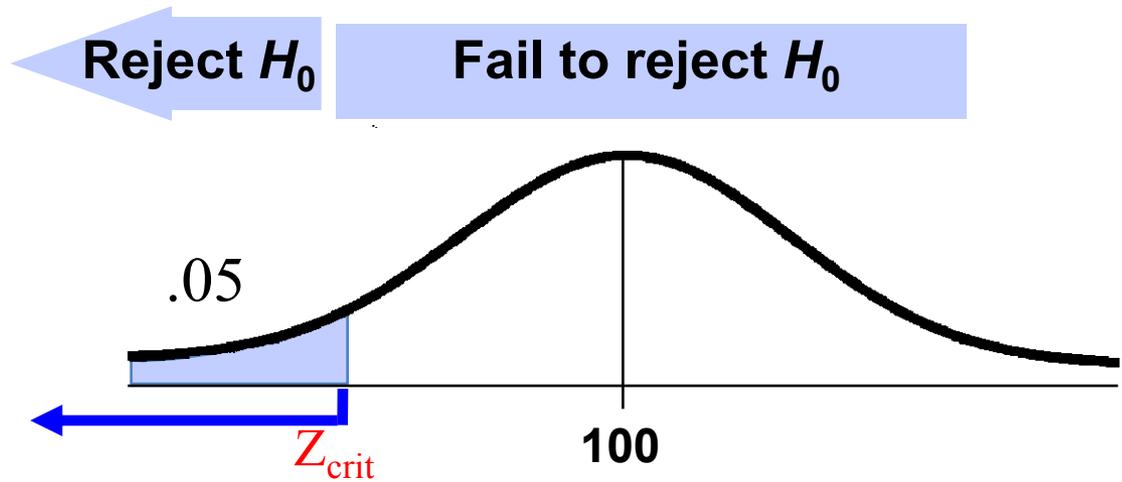
b) Two – tailed test

$H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$ (does not equal or different from)

One- vs. Two-Tailed Tests

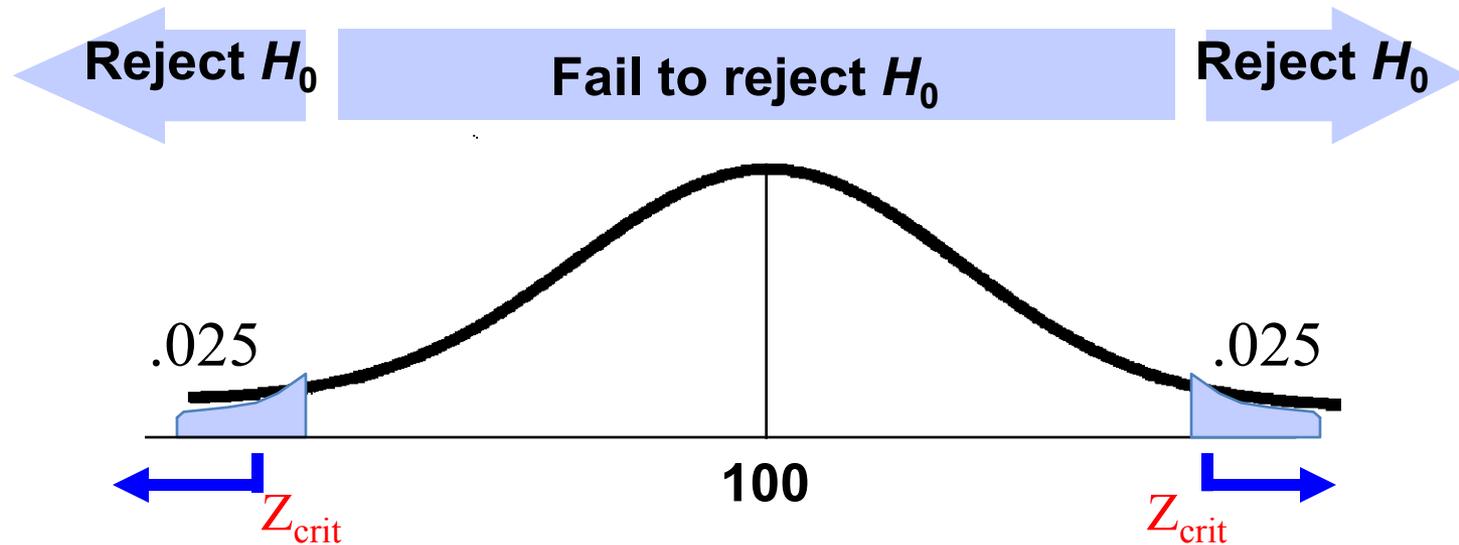
- In theory, you should use one-tailed when
 1. Change in opposite direction would be meaningless
 2. Change in opposite direction would be uninteresting
 3. No opposing theory predicts change in opposite direction
- By convention/default in the social sciences, two-tailed is standard
- Why? Because it is a more strict criterion. A more conservative test.

One tail



Values that differ "significantly" from 100

Two tail



Values that differ significantly from 100

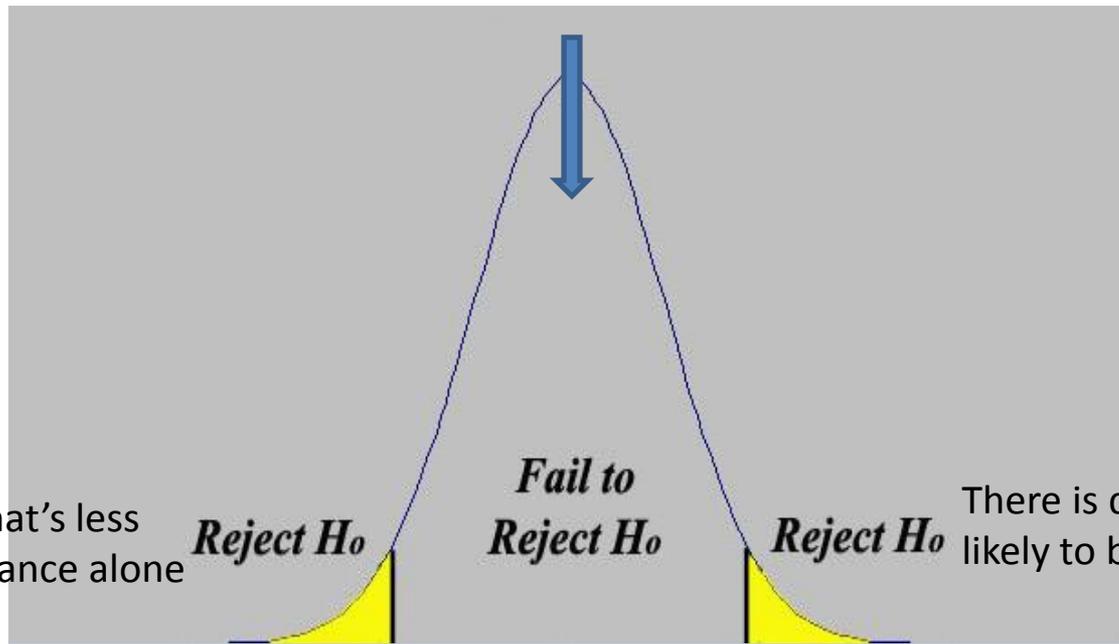
Step 5

- Compare the test statistics value with the critical value of rejection.

Does Z-value = or \neq value from the table

Step 6

- Decide whether to reject the null hypothesis and confidence statement.



There is difference that's less likely to be due to chance alone

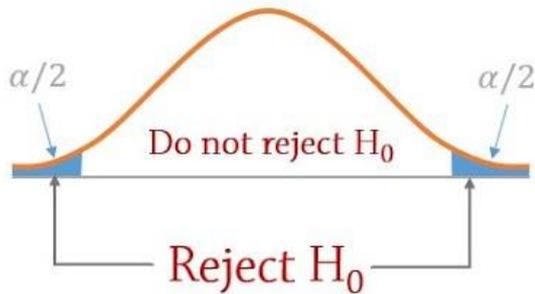
There is difference that's less likely to be due to chance alone

Hypothesis Testing

Two-tailed

$$H_0: \mu = 23$$

$$H_1: \mu \neq 23$$

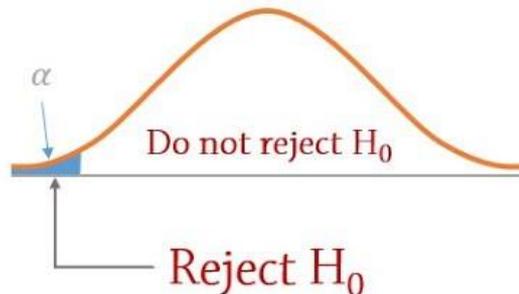


One-tailed

Left-tailed

$$H_0: \mu \geq 23$$

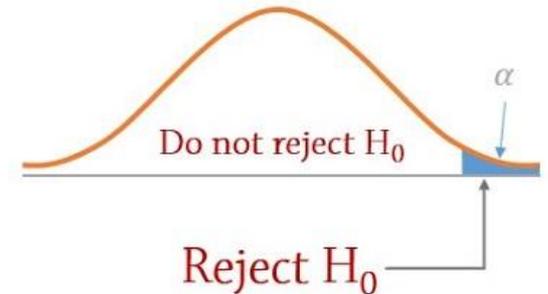
$$H_1: \mu < 23$$



Right-tailed

$$H_0: \mu \leq 23$$

$$H_1: \mu > 23$$



Rejection Rule

Lower (left)-tailed test

Hypotheses

$$H_0: \mu = \mu_0$$

vs $H_1: \mu < \mu_0$ (less than or smaller than)

Rejection Rule

Reject H_0 at a level of significance α if:

- i. $Z < -Z_{1-\alpha}$ (In case of using Z-distribution)
- ii. $t < -t_{(\alpha, n-1)}$ (In case of using t-distribution)

Rejection Rule

Upper (right)-tailed test

Hypotheses

$$H_0: \mu = \mu_0$$

vs $H_1: \mu > \mu_0$ (more than or greater than)

Rejection Rule

Reject H_0 at a level of significance α if:

- i. $Z > Z_{1-\alpha}$ (In case of using Z-distribution)
- ii. $t > t_{(\alpha, n-1)}$ (In case of using t-distribution)

Rejection Rule

Two-tailed test

Hypotheses

$$H_0: \mu = \mu_0$$

vs $H_1: \mu \neq \mu_0$ (does not equal or different from)

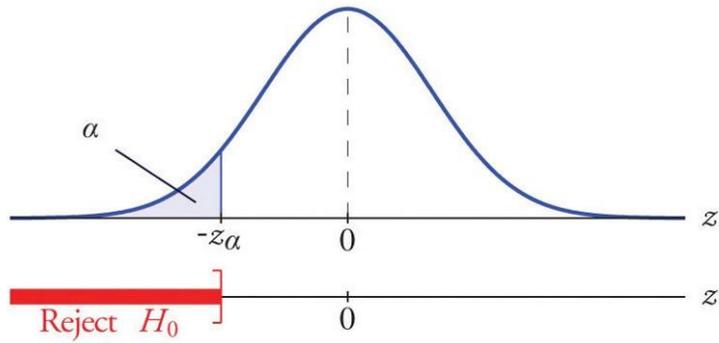
Rejection Rule

Reject H_0 at a level of significance α if:

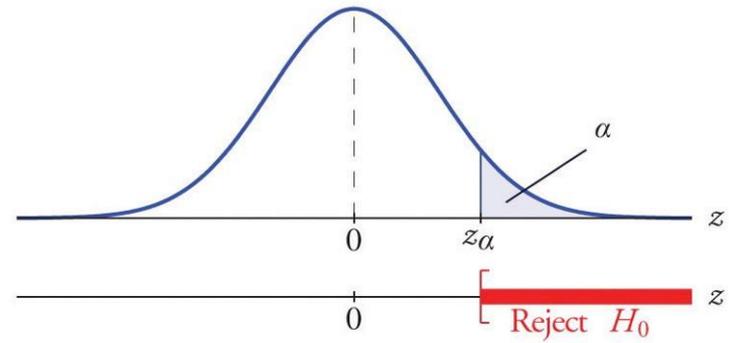
i. $|Z| > Z_{1-\frac{\alpha}{2}}$ (In case of using Z-distribution)

ii. $|t| > t_{\left(\frac{\alpha}{2}, n-1\right)}$ (In case of using t-distribution)

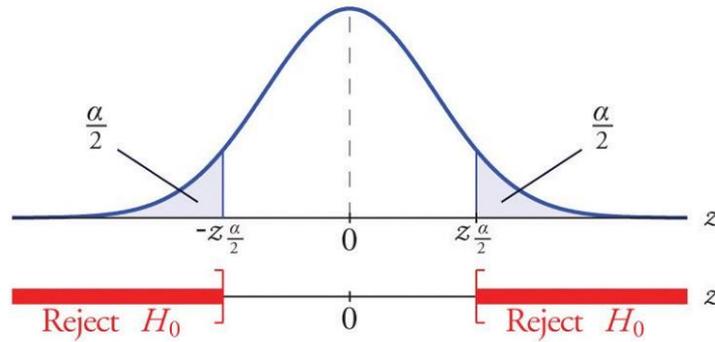
$H_a : \mu < \mu_0$



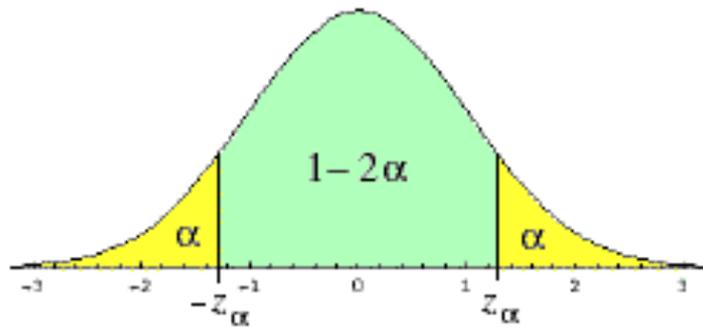
$H_a : \mu > \mu_0$



$H_a : \mu \neq \mu_0$



Common Critical Values



$\alpha = \text{tail area}$	central area = $1 - 2\alpha$	z_α
0.10	0.80	$z_{.10} = 1.28$
0.05	0.90	$z_{.05} = 1.645$
0.025	0.95	$z_{.025} = 1.96$
0.01	0.98	$z_{.01} = 2.33$
0.005	0.99	$z_{.005} = 2.58$

α	$1 - \alpha$	$z_{1-\alpha}$
0.10	0.90	$z_{0.90} = 1.28$
0.05	0.95	$z_{0.95} = 1.645$
0.01	0.99	$z_{0.99} = 2.33$

Example Weight

Salem believes that his “true weight” is 72kg with a standard deviation of 3kg.

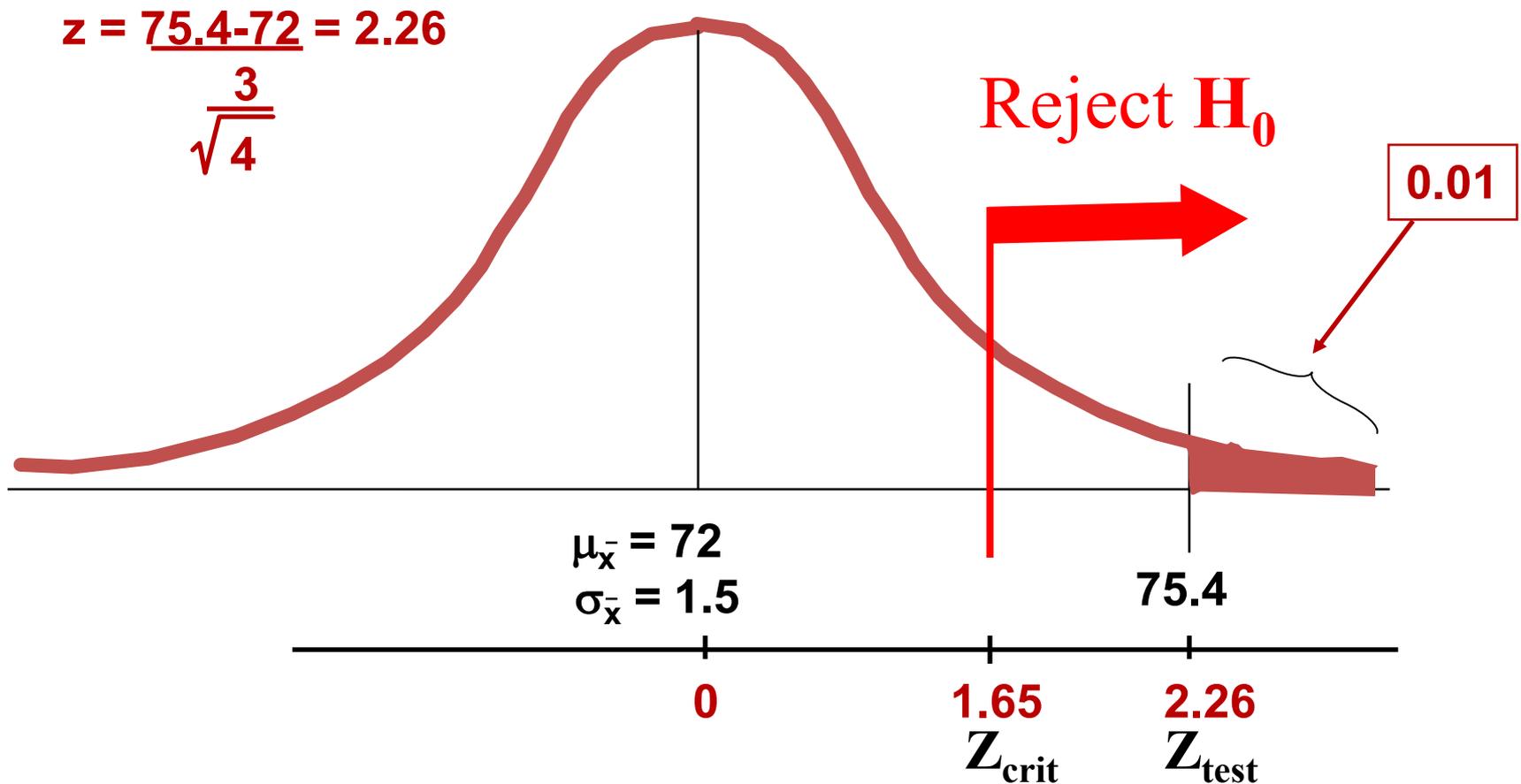
Salem weighs himself once a week for four weeks. The average of these four measurements is 75.4kg.

Are the data consistent with Salem’s belief?

Example Weight

1. $H_0: \mu = 72$ $H_1: \mu > 72$ This is a one tail test
2. $\alpha = 0.05$
3. $\mu > 72$ (one tail test)
4. $Z_{\text{crit}} = Z_{\text{Reject } H_0}$ if $z \geq 1.645$
5. $Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{75.4 - 72}{3/\sqrt{4}} = 2.26$ $P(Z > 2.26) = .012$
6. Since $2.26 > 1.645$, we Reject H_0 . There is a statistically significant evidence at $\alpha = 0.05$ to show that the mean weight measured is higher than his original belief about his weight. The chance that the measured weight and initial (belief) weight means are different due to chance only is less than 5%.

Example Weight illustrated



Example

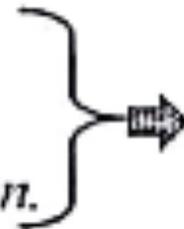
Researchers are interested in the mean age of a certain population. They are wondering if the **mean age** is more than **25** years. Assuming that the **population is normally** distributed with **variance** equal to **20**. A **random sample** of **10** individuals drawn from the population of interest. From this sample, a **mean** of **27** is calculated. Construct the proper hypothesis, test your hypothesis, and then state the proper conclusion? Use $\alpha = 0.05$ to test the hypothesis?

Solution

We have:

1- Normal distribution.

2- The standard deviation σ is known.



Then we will use the standard normal distribution (Z).

$$n = 10 , \mu_0 = 25 , \bar{X} = 27 , \sigma = \sqrt{\sigma^2} = \sqrt{20} = 4.472 , \alpha = 0.05$$

Example

Hypotheses

$$H_0: \mu = \mu_0 \longrightarrow H_0: \mu = 25$$

$$\text{vs } H_1: \mu > \mu_0 \longrightarrow H_1: \mu > 25 \text{ (more than)}$$

Rejection Rule

Reject H_0 at a level of significance $\alpha = 0.05$ if:

$$Z > Z_{1-\alpha}, Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = \mathbf{1.645}$$

Test statistics (calculated value)

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{27 - 25}{4.472 / \sqrt{10}} = \mathbf{1.414}$$

Continued

Decision

We get $Z = 1.414 < Z_{1-\alpha} = 1.645$

Then the rejection is not satisfied and therefore the decision will be do not reject (accept) H_0 at $\alpha = 0.05$ and therefore we conclude that the mean age is 25 years, that is, $\mu = 25$ years

Example

The **mean** maximum oxygen uptake for a sample of **242** women was **32.3** with a **standard deviation** of **12.14**, we wish to know if, on the basis of the data, can we conclude that the **mean** score for a population of such women is **smaller than 33**? Use $\alpha = 0.01$ to test the hypothesis?

Solution

We have:

1- Unknown distribution (population).

2- The standard deviation σ is unknown ($S = 12.14$).

3- The sample size (n) is large ($n = 242 > 30$).

Then we will use
the standard normal
distribution (Z).

$$n = 242 , \mu_0 = 33 , \bar{X} = 32.30 , S = 12.14 , \alpha = 0.01$$

Continued

Lower (Left) -Tailed Test

Hypotheses

$$H_0 : \mu = 33$$

vs $H_1 : \mu < 33$ (smaller than).

Rejection Rule

Reject H_0 at level of significance $\alpha = 0.01$ if :

$$Z < -Z_{1-\alpha}$$

Test Statistic (Calculated Value)

$$Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

$$\Rightarrow Z = \frac{32.30 - 33}{12.14 / \sqrt{242}} = -0.897$$

Decision

We get $Z = -0.897 > -Z_{1-\alpha} = -2.33$

Then the rejection rule is not satisfied and therefore the decision will be do not reject (accept) H_0 at $\alpha = 0.01$ and therefore we conclude that the mean score for a population of such women is 33, that is, $\mu = 33$.

Critical Value (Tabulated Value)

$$\begin{aligned} Z_{1-\alpha} &= Z_{1-0.01} = Z_{0.99} = 2.33 \\ \Rightarrow -Z_{1-\alpha} &= -Z_{1-0.01} = -Z_{0.99} = -2.33 \end{aligned}$$

Example

The body mass index (BMI) of a **group** of **14** healthy adult males has a **mean of 30.5** and a **standard deviation of 10.6392**, can we conclude that the **mean BMI of the population** is equal to **36** assuming that the population is normally distributed? Use **$\alpha = 0.1$** to test the hypothesis?

Solution

We have:

1- *Normal distribution (Normal population).*

2- *The standard deviation σ is unknown*

($S = 10.6392$).

3- *The sample size (n) is small ($n = 14 < 30$).*

➡ Then we will use the t-distribution.

$$n = 14 , \mu_0 = 36 , \bar{X} = 30.5 , S = 10.6392 , \alpha = 0.10$$

Two -Tailed Test

Hypotheses

$$H_0 : \mu = 36$$

vs $H_1 : \mu \neq 36$ (does not equal).

Rejection Rule

Reject H_0 at level of significance $\alpha = 0.10$ if :

$$|t| > t_{\left(\frac{\alpha}{2}, n-1\right)}$$

Test Statistic (*Calculated Value*)

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

$$\Rightarrow t = \frac{30.5 - 36}{10.6392/\sqrt{14}} = -1.934$$

Critical Value (*Tabulated Value*)

$$\begin{aligned} & t_{\left(\frac{\alpha}{2}, n-1\right)} \\ &= t_{\left(\frac{0.10}{2}, 14-1\right)} \\ &= t_{(0.05, 13)} \\ &= 1.771 \end{aligned}$$

Decision

We get $|t| = |-1.934| = 1.934 > t_{\left(\frac{\alpha}{2}, n-1\right)} = t_{(0.05, 13)} = 1.771$

Then the rejection rule is satisfied and therefore the decision will be **reject** H_0 at $\alpha = 0.10$ and therefore we conclude that the mean BMI of the population is not equal to 36, that is, $\mu \neq 36$. In other words H_1 is accepted at $\alpha = 0.10$.